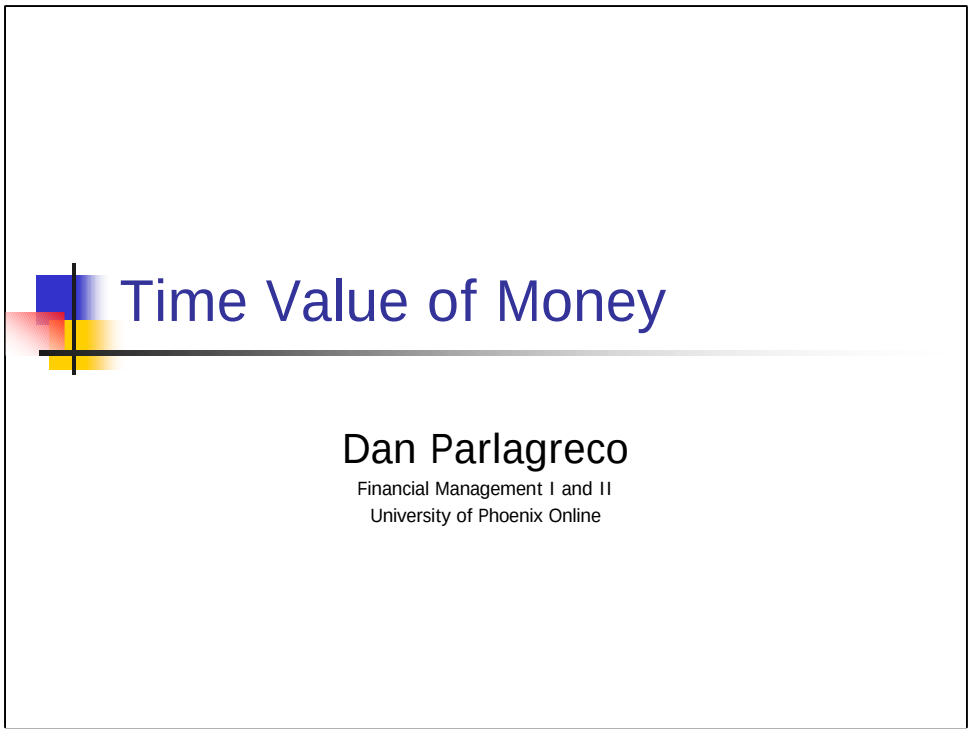
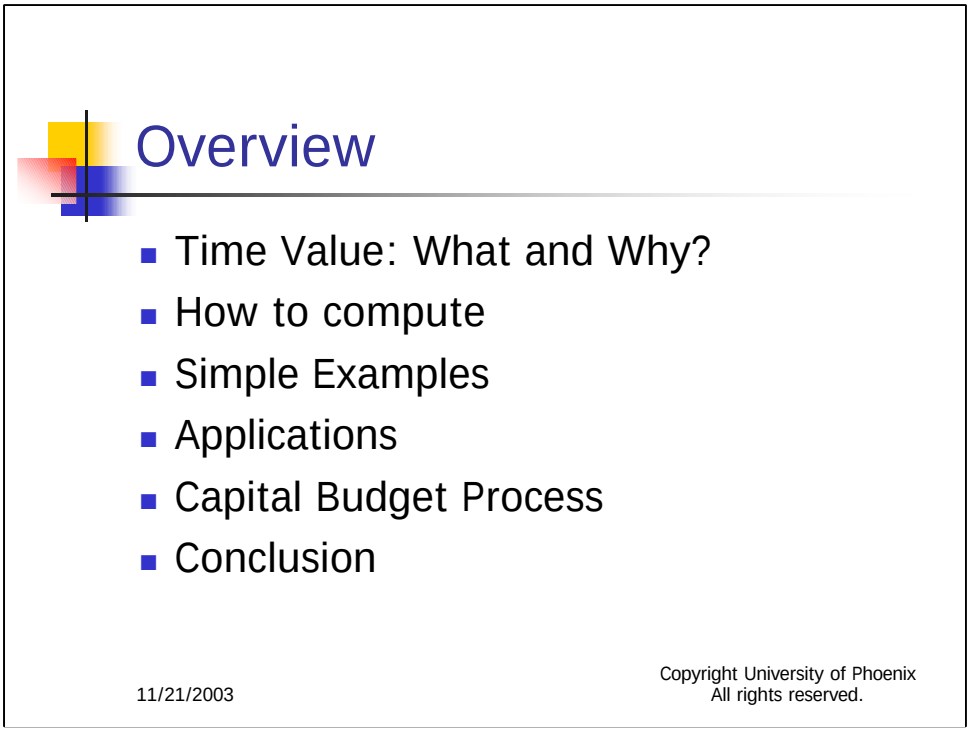
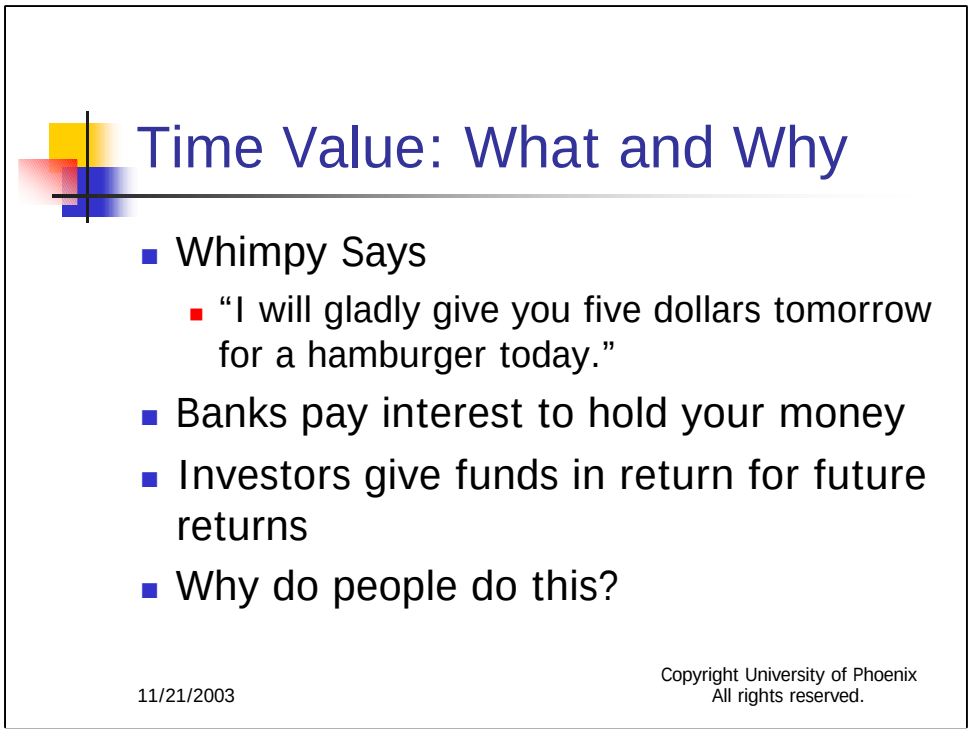


Welcome to a brief introduction to the concept of the time value of money.

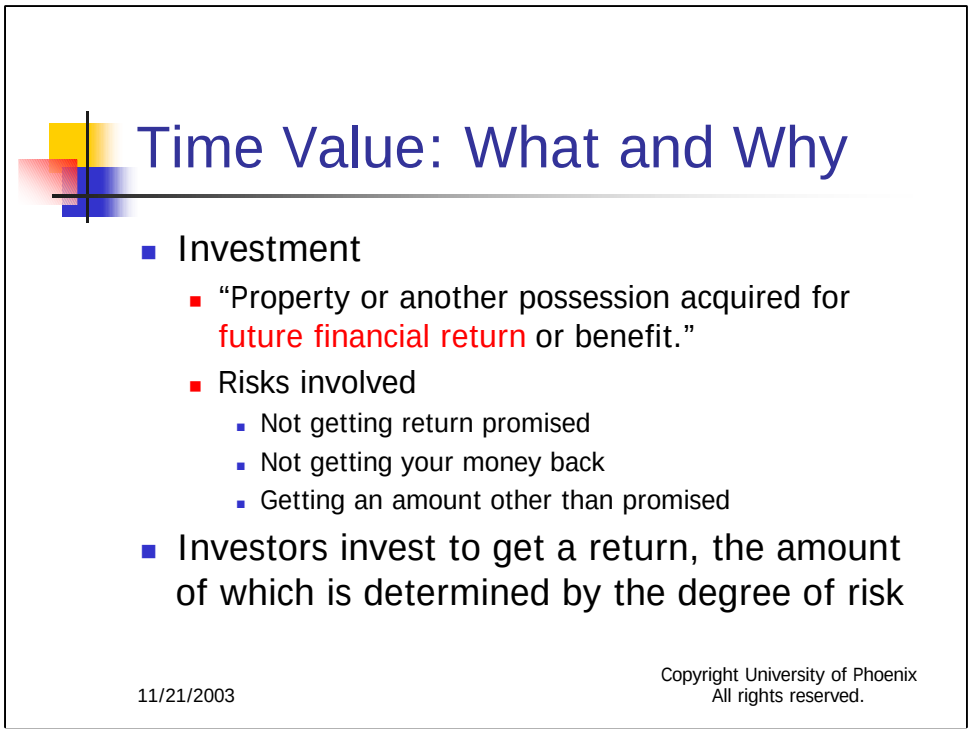


During this presentation, we will explore what the time value of money is, how we compute it and how we use it in simple examples. We will look beyond the simple to real life applications, where time value of money concepts are typically used. One of the main areas of application for time value of money is capital budgeting analysis, and we will consider some of the aspects associated with project evaluation as an illustration of the concepts we are considering. We will conclude with a few observations about money and its value over time.



Popeye was one of my favorite cartoons as a kid. In the cartoon, a rather portly character, Whimpy, often had the line "I will gladly pay you $5.00 tomorrow for a hamburger today." This was meant to be funny, because at the time a hamburger could be had for about $0.50. It is also an illustration as to why money has time value, Whimpy wanted his hamburger now, and was willing to forgo the possibility of ten hamburgers in the future to get it. (another part of the joke, however, was that Whimpy would never pay, another reason why its better to have money now then later).

More realistically, we know that banks are willing to pay us interest and that investors are willing to risk their money in bonds and stocks in return for the expectation of more money later. Why do it? The simple answer is that investors will invest funds when the expectation of future returns makes them indifferent between the funds they have now and the funds they expect to receive in the future.

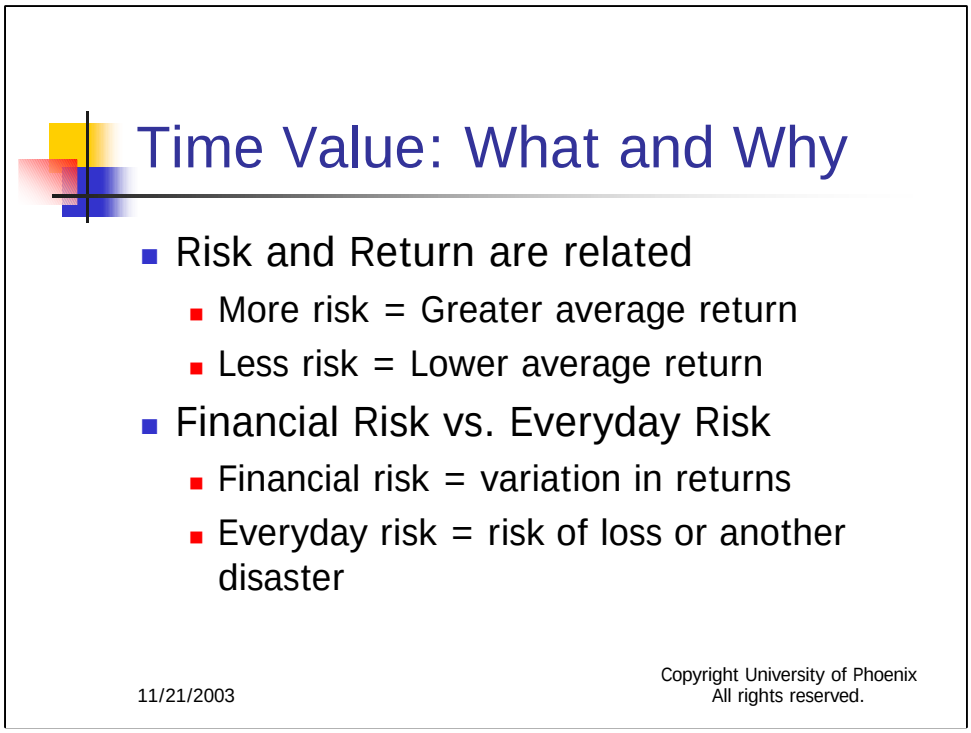


So what is an investment, anyway?

The dictionary defines investment as "property or another possession acquired for future financial return or benefit," meaning that its something you are not using now in the expectation that you will get to use more in the future.

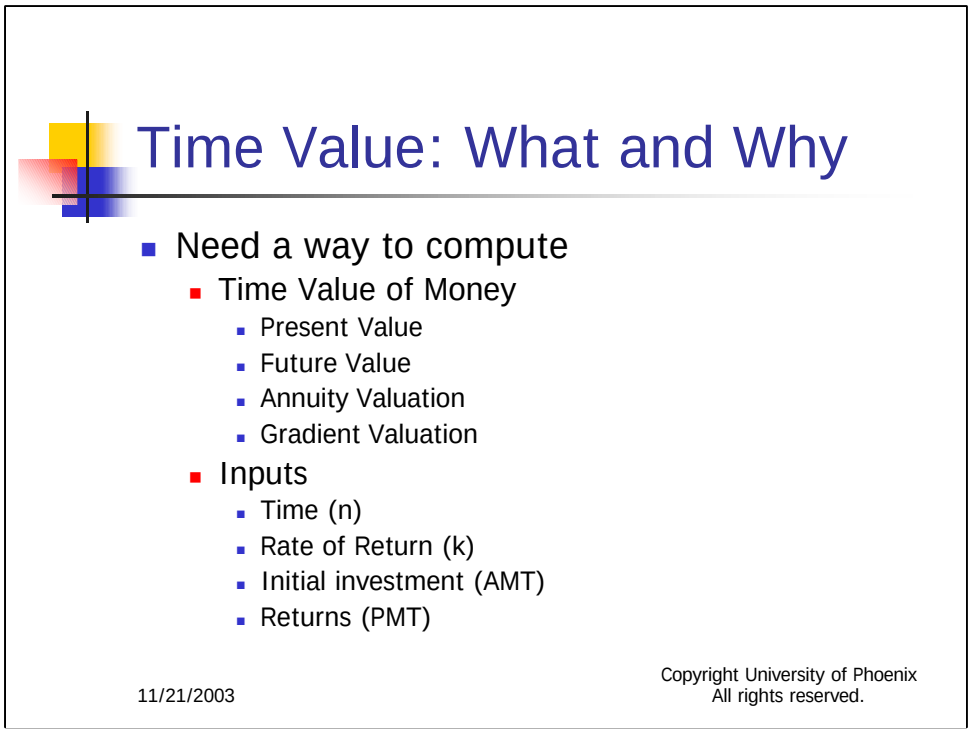
Investments do have risks, which increase the required return of investors. Some of the risks are obvious, like not getting the return you expected or not getting your money back. Not so obvious is the risk that you might get a lot more or less than you expect. Variation in returns is the financial definition of risk.

Investors invest to get a return, the amount of which is determined by the degree of risk they are undertaking.



This means risk and return are related. The higher the risk, the greater the return you will need. A low risk investment, however, gives a near certain return and therefore the investor need be compensated only a little since his risk is low. This is why the banks pay 1% on a passbook account, which is guaranteed by the government and which will almost certainly pay the rate indicated. It is also why risky companies have to pay 10% and more to get funds; investors know there is a chance they will not get the amount promised and even a chance they might loose their money.

Everyday risk and financial risk are different. Everyday risk refers usually to a risk of a loss or injury. In other words, you risk your life, your limb, your house, etc… Financial risk means variation in returns. A risky investment in a financial sense has a high variation in return which leads to investors bidding the price to a level where the average expected return is higher than if the return were certain, even if the certain return were less than the average return expected.



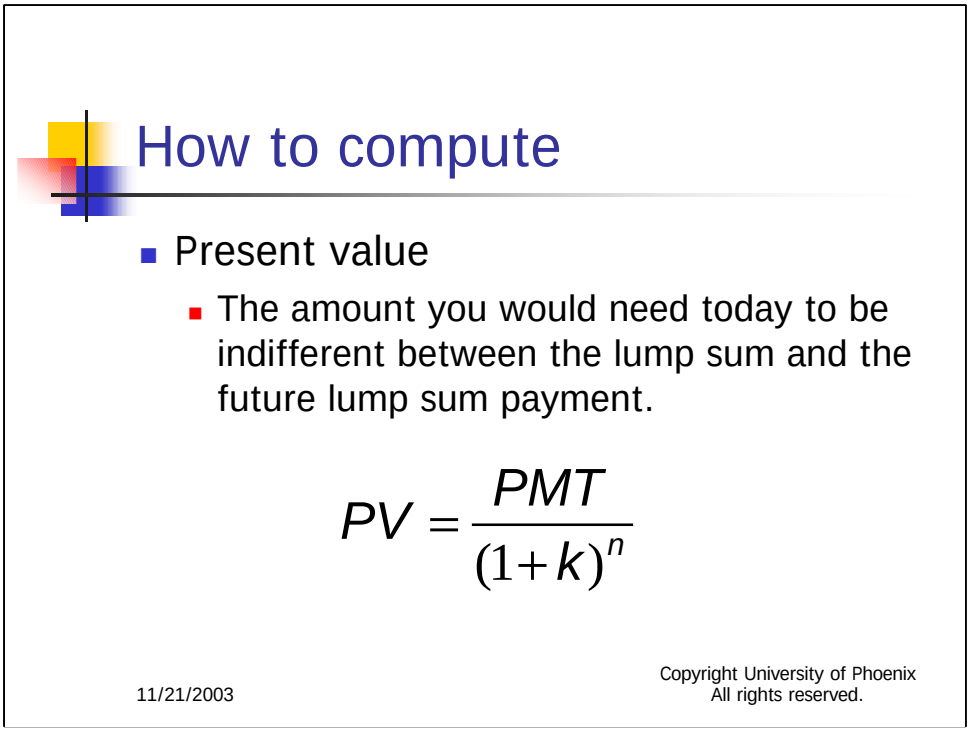
So how do we compute this? We use time value of money concepts. Given the time in number of periods (n), rate of return expected (k), sum involved (AMT) and return (PMT), you can solve for the following amounts:

Present Value: the amount required to make an investor indifferent between a sum today and another larger sum in the future.

Future Value: the amount required in the future to make the investor indifferent between that sum and a smaller sum today.

Annuities are equal payments spread over time, and these can be valued in relation to lump sums today (present value) or at some future time (future value).

Gradients are equally increasing annuity payments, and are beyond the scope of our present discussion.



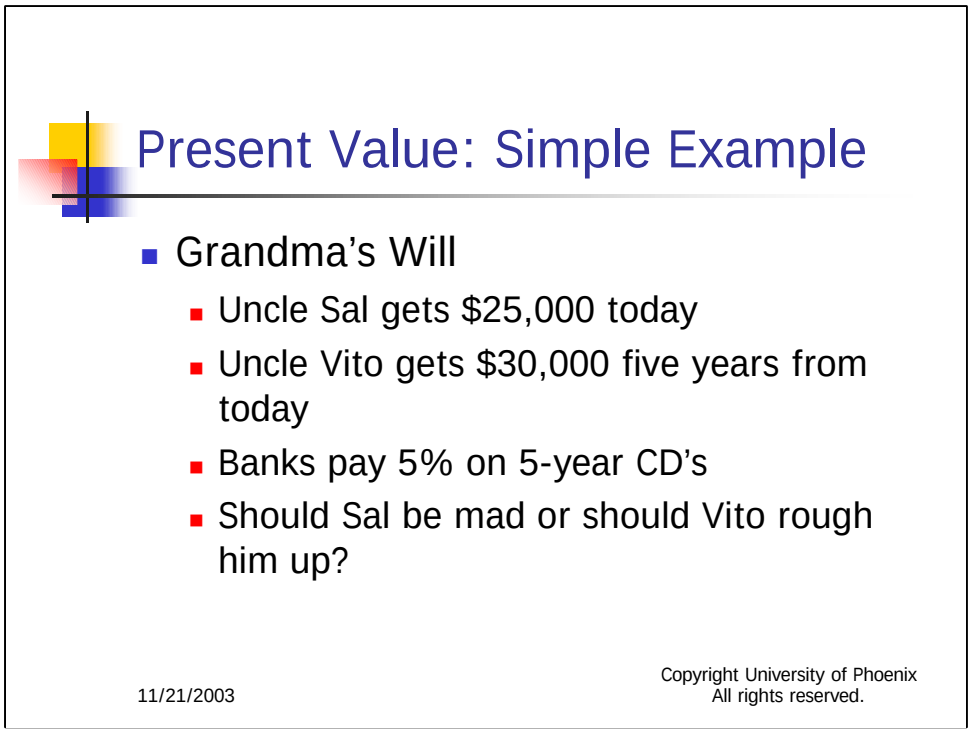
To compute the present value of a future sum (PMT), just divide by the quantity one plus the interest (or discount) rate and raise the rate to the nth power where n is the number of periods.

A word on the math:

Raising a sum to a power is the same as multiplying it out n times (if n is the power).

So $(1+k)^3$ is the same as (1+k)(1+k)(1+k)

Be sure that k and n are for comparable periods. If periods are months, you need to use a monthly interest rate for k. So if an annual rate is given, you would divide by 12 to make it into a monthly rate to match the monthly periods in that case.



Lets look at an example.

Uncle Sal gets $25K now, so the present value of that is easily seen to be $25,000.00

Uncle Vito gets $30,000 five years from now and banks pay 5% annual interest.

Question is, who got the short end of Grandmas stick?

To answer this, we would compute the present value of Uncle Vitos $30K to be received 5 years from now at a discount rate of 5%, since Vito could put a sum in the bank today and get that amount back.

Present Value: Simple Example

- PV of Sal's "take"

$$PV = 25{,}000\left(\frac{1}{(1+0.05)^{0}}\right) = \$25{,}000.00$$

- PV of Vito's "take"

$$PV = 30{,}000\left(\frac{1}{(1+.05)^{5}}\right) = \$23{,}505.78$$

11/21/2003

Here Grandma's favorite is revealed. As we see, Sal should be happy, until Vito breaks his legs, that is…

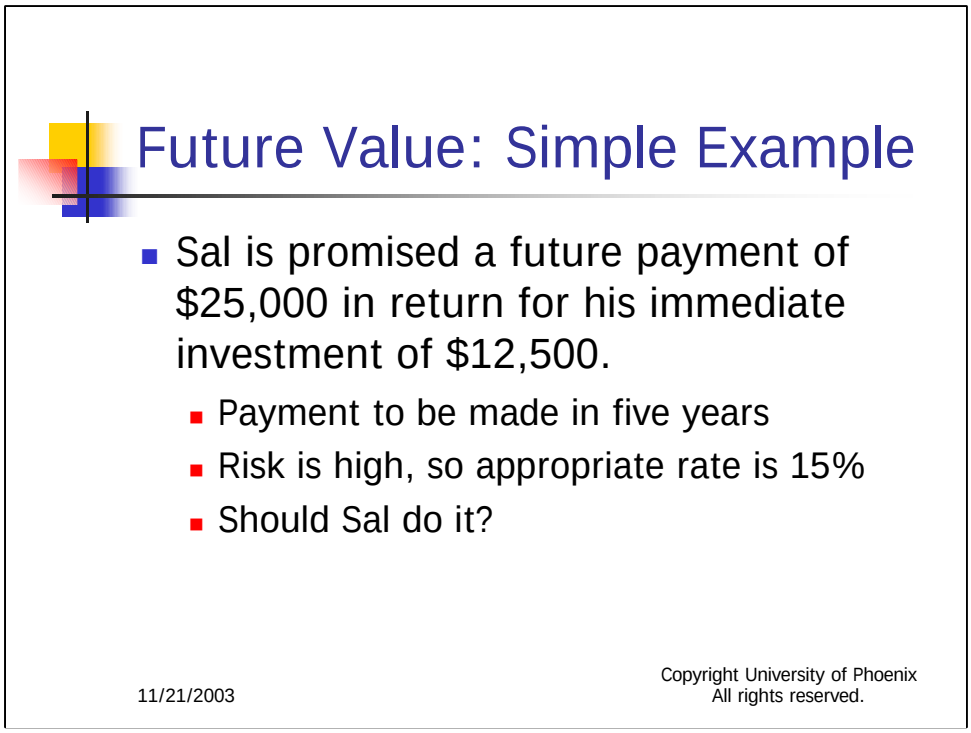
How to Compute

- Future Value
  - The amount you would need at some time in the future to be indifferent between a different (smaller) amount to be received today and that future amount.

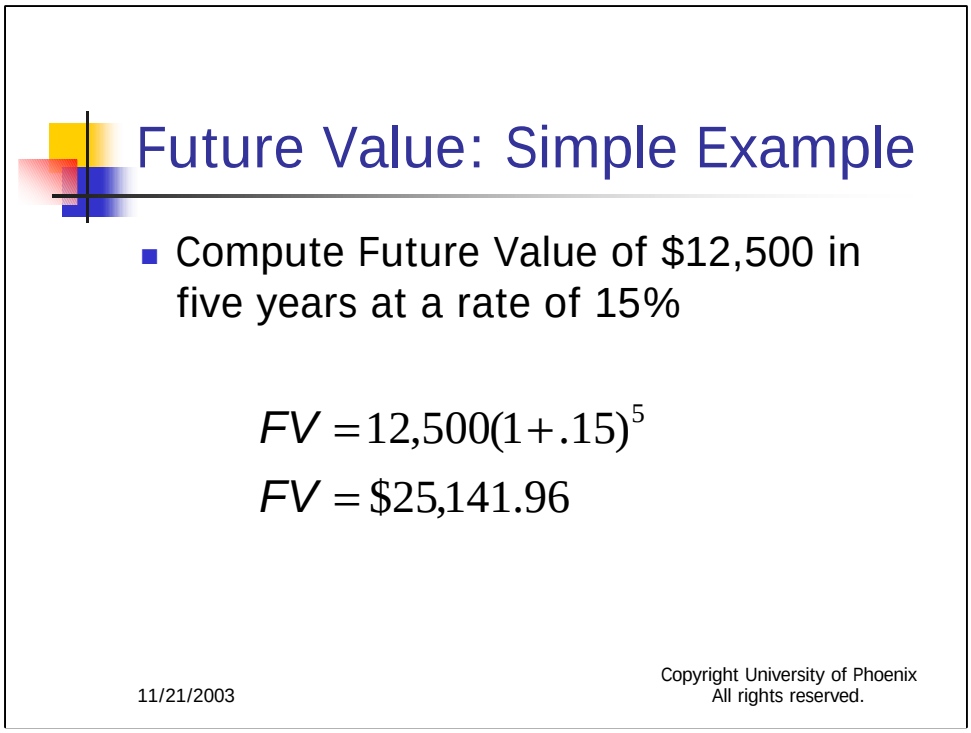
$$FV = PV(1+k)^n$$

11/21/2003

Future value is the exact same computation as present value, but in reverse. Al the same guidelines apply to this simple calculation.

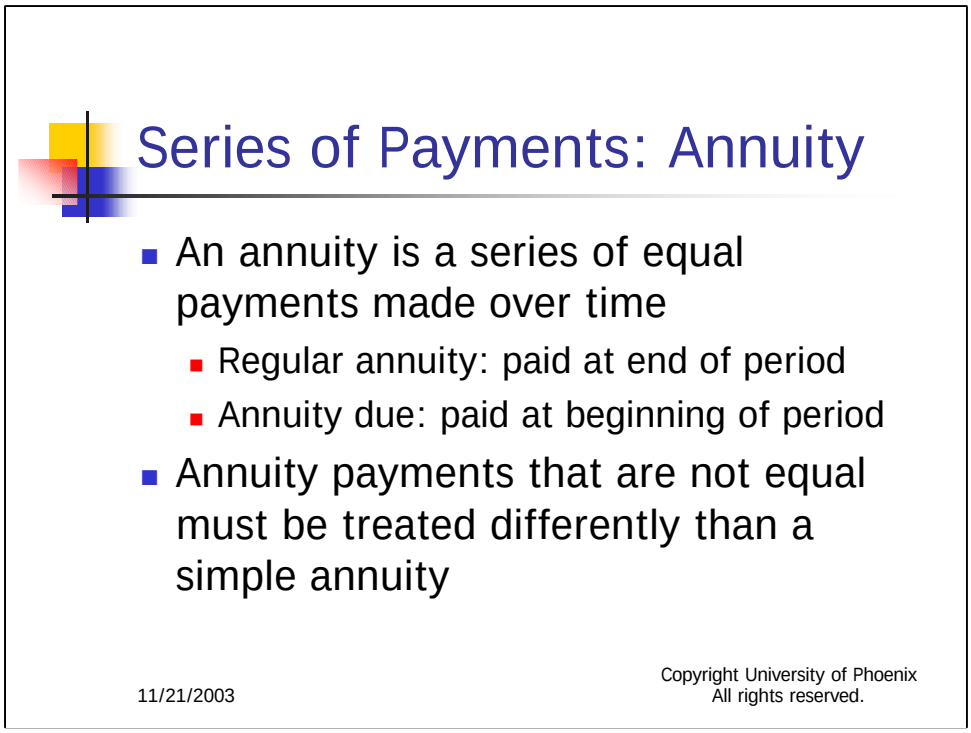


So if Sal is promised $25K in five years if he would just give Vito $12.5K now, and Sal thinks a discount rate of 15% is appropriate, should he take the deal or pass on it while he recuperates in the hospital?



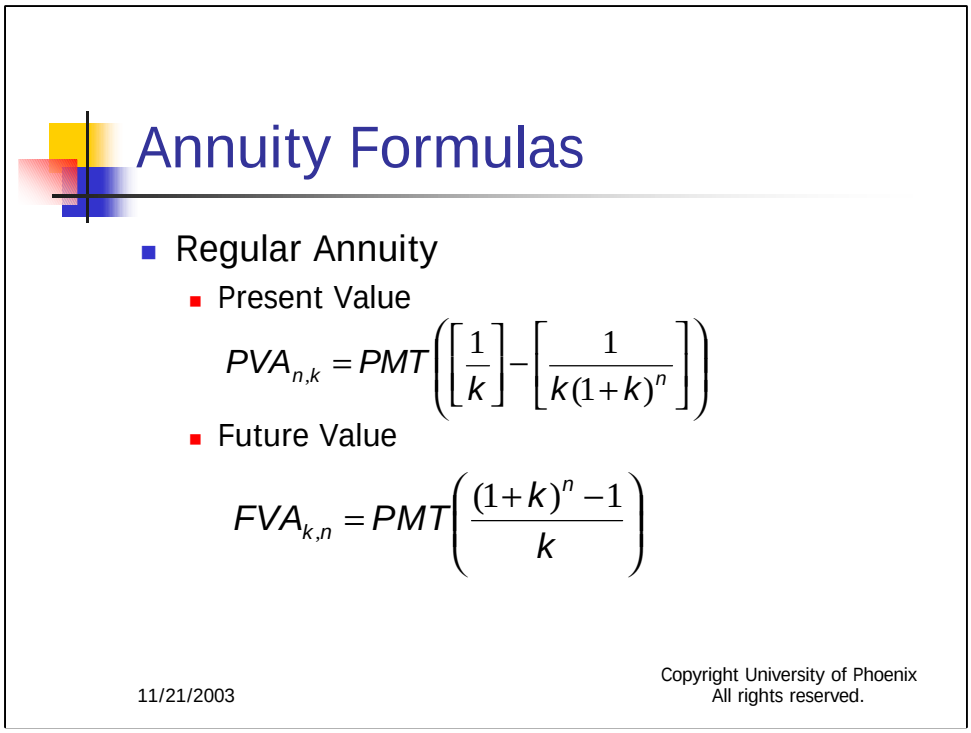
Not being a fool, Sal computes that the future value of his $12.5K at 15% is just over $25,141.00, meaning he should pass up Vito on his fine offer and opt for another investment.

The minimum payback that Sal would require to risk his money on Vito is $25,141.96, meaning any proposed payback less than that would be unacceptable to Sal.



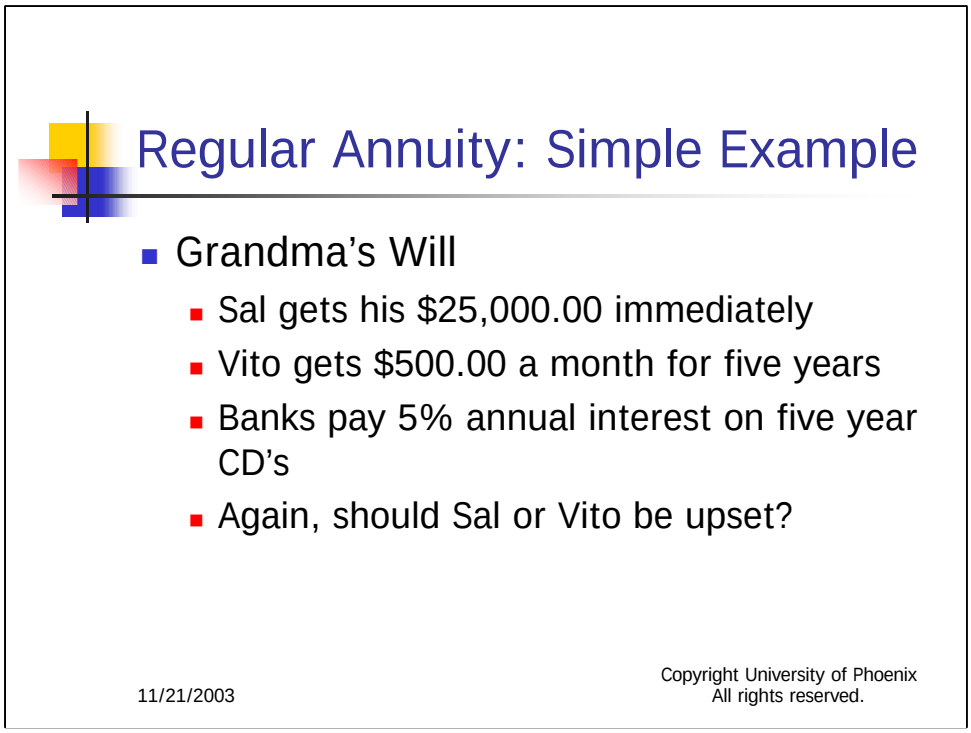
Shifting gears a bit, we see that an annuity is a series of equal payments. If the payment is made at the end of the period, its called a regular annuity; if at the beginning it's called an annuity due.

Annuity payments that are not equal must be treated as unequal cash flows and each year must be handled separately.



The formulas here can be used to compute the present and future values of any annuity given the periodic interest rate k, the number of periods n, and the annuity payment amount PMT.

Note that if n is infinite, the annuity is called a perpetuity, and the formula for the present value reduces to PMT/k (with k in periodic form to match the frequency of receipt of PMT).



Lets look at another version of Grandma's Will and see if Sal or Vito is the better man.

Sal gets his usual $25K, but Vito has now talked Grandma into giving him $500 per month for five years.

The task is to value Vitos new inheritance at the same time Sal gets his, which is now.

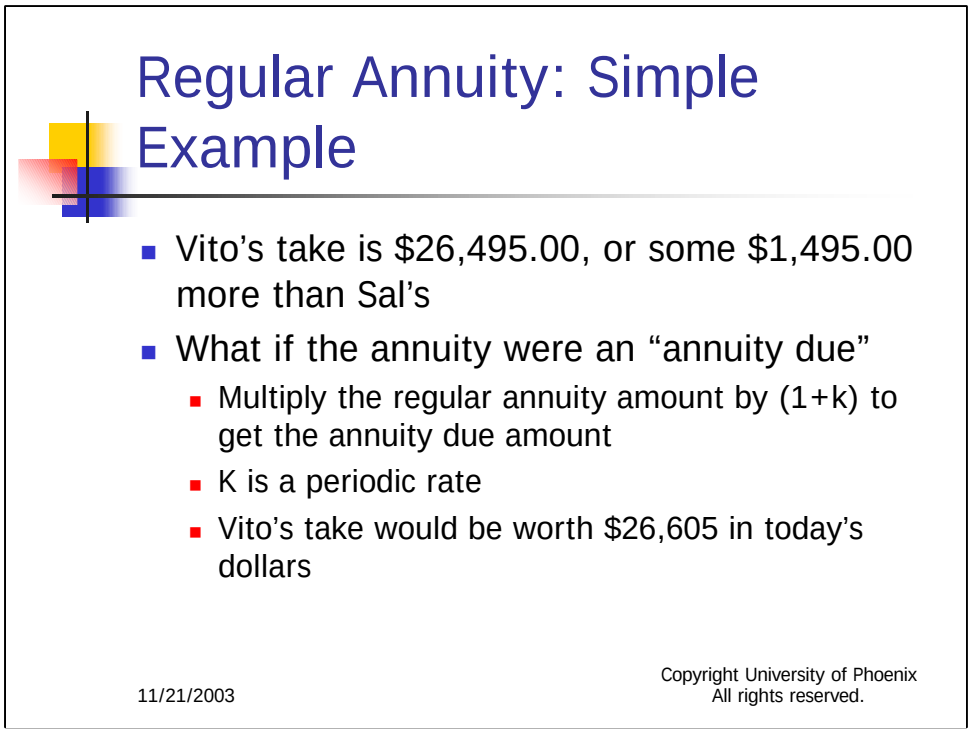
Regular Annuity: Simple Example

- Sal's "take" is still $25,000.00
- Vito has an annuity for five years.
  - Five years = 12 x 5 or 60 months
  - Monthly interest is 0.05/12 or 0.004167

$$PVA_{60,0.004167} = 500\left(\left[\frac{1}{.004167}\right] - \left[\frac{1}{0.004167(1+0.004167)^{60}}\right]\right)$$

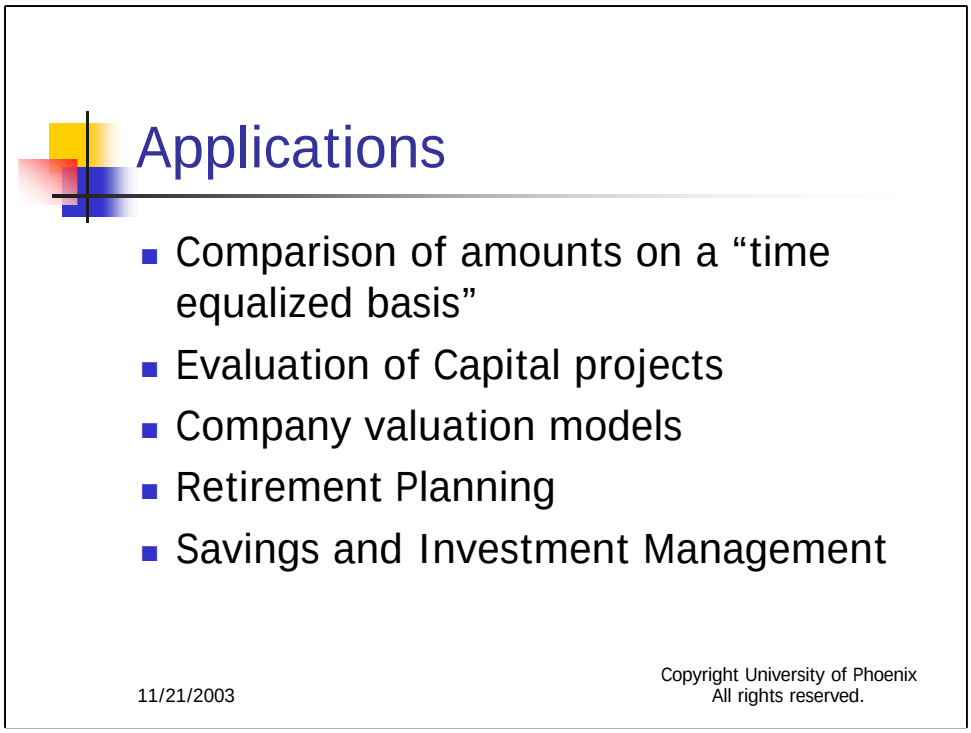
11/21/2003

Using the formula, we know that 5 years is 60 months and that 5% equals a 0.004167 monthly interest rate (0.4167% expressed as a decimal).

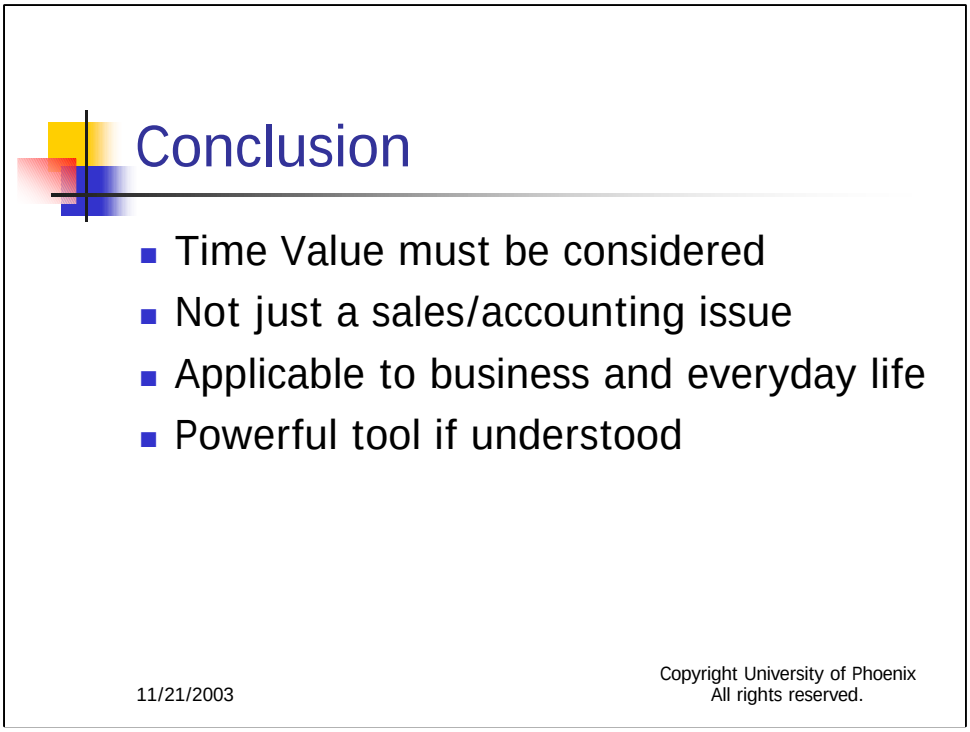


Solving for the present value, we see Vito now is in the drivers seat, getting some $1,495.00 more than Sal. Note that the sum of the payments is still $30,000 like the first version of the will, but that smooth talking Vito has tricked Grandma into giving him a lot more than the $30K he would have got as a lump sum five years in the future.

If the annuity were an annuity due, Vito would be in even better shape. The value of an annuity due is computed by taking the value of the regular annuity and multiplying by (1+k).



Here are some common applications of these techniques, including capital budgeting, valuation of companies for mergers, retirement planning, amortization of loans, and savings and investment management.



So we have seen why money has a time value, how to compute what that value is, and have touched on how it might impact an everyday situation. Clearly, the time value of money is a powerful concept for those who understand it.