Finance 476

Week 5 Lecture Notes

Objectives

- Understand options, what they are and how to price them.

- Understand international finance and interest rate parity.

Important Formulas

Black Scholes Option Pricing Model

$$V = P[N(d_1)] - Xe^{-k_{rf}t}[N(d_2)]$$

$$d_1 = \frac{\ln\left(\dfrac{P}{X}\right) + [k_{rf} + \left(\dfrac{\sigma^2}{2}\right)]t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Where V is the value of the option in dollars, P is the current price of the underlying security, X is the strike price, N(di) is the probability that an event will occur less than di in a normal distribution (see table in back of book for values of N), e is the exponential function equal to 2.7183, krf is the risk free rate, t is the time to expiration of the option and sigma squared is the measure of the market variance.

Interest Rate Parity Model

$$\frac{FwdRate}{SpotRate} = \frac{(1 + k_h)}{(1 + k_f)}$$

Where kh is the home interest rate and kf is the foreign interest rate. Both the forward and spot rates are expressed as amount of home currency per unit of foreign currency.

Discussion

Options are the right but not the obligation to purchase or sell a security at a given price at a fixed date in the future. The right to sell a security at a given price is called a put option and the right to buy a security is a call option. Combinations of put and call options along with ownership or non-ownership of

the underlying security can result in complex positions called hedging strategies. For example, an investor can write a put option and have the stock at his brokerage that would back the sale at the future date. This would be called a covered put option. This means that if the purchaser of the put option decides to execute his right to sell the stock at the given price, he can do so and the investor would have the shares to sell to cover the put.

More common is the covered call option. In the covered call, the investor writes a call option and also has the stock in his brokerage account. If the stock price goes high enough to make the option "in the money" then the investor can sell the stock in his brokerage account on the execution date and escape without massive losses.

Naked call options, for example, would be very risky for the investor writing (issuing) the option. In this case, the investor writes the option without having the security in his account. Since the amount by which a stock can increase is unlimited, the potential for loss in a naked call is unlimited for the writer of the option.

For the buyer of an option, the limit of the loss is the price paid for the option.

To determine the fair price of an option, it is common to use the Black Scholes Option Pricing Model noted above. To illustrate the application of this complex model, lets consider the following example:

Security: ABD
Current Price: $45.00 per share
Market Coefficient of Variation: 0.10
Strike Price: $55.00
Time to Expiration of Option: 3 months
Risk Free Rate: 6%

First solve for d1 from the equation noted above:

$$d_1 = \frac{\ln\left(\frac{45}{55}\right) + [0.06 + (.10/2)](.25)}{.3162\sqrt{.25}}$$

Note that 3 months is equal to 3/12 or 0.25 years and that the market sigma is equal to the square root of the market coefficient of variation.

Solving this equation yields a value of d1 of −1.0953.

Solving now for d2 is easy:

$$d_2 = -1.0953 - .3162\sqrt{.25}$$

which yields d2 as –1.2534

Now that we have d1 and d2, we can find the area under the normal distribution using table A-5 in Appendix A of the book. So N(d1) is 0.5 - 0.3621 or 0.1379 and N(d2) is 0.5 - 0.3944 or 0.1056.

We can now plug these figures into the basic Black Scholes model to obtain the price of the option:

$$V = 45(.1379) - 55e^{-(.06)(.25)}(.1056)$$

This gives an option value of $0.484.

Mathematical Notes: Easy mistakes to make.

To help avoid mistakes, consider the following:

1. In the equation for d1, the function ln is used. This is not the same as log. The function ln is the natural logarithm, which is based on the constant e. The function log is the base 10 logarithm. If you use the wrong one, your answer is wrong.

2. Solve for d1, then d2 then V.

3. When looking up the value of N(x) remember that the table is split up into gross and fine gradations. First find the number closest to the value you are looking for (ignoring the sign convention) along the side. Then read over to the column corresponding to the value you are finding. Consider the number 1.26 as the combination of 1.2 and 0.06. You would find the row for 1.2 in the table and read over to the 0.06 column to find N(1.26).

4. Once you have the value from the table, you can find the value of N(d1) by considering that you have to add 0.50 in either case. This is necessary because the table only is for half the normal curve and to get the whole curve you have to add back the other half.

Interest Rate Parity

It is important to consider the effect of local interest rates on currency exchange rates. When a country maintains a low rate of interest, it is very possible that inflation will result. Low interest rates make people in the country look outside that country to find higher returns. As they move their money outside the country, they need to trade the local currency for the foreign currency.

Say the Euro is trading at 0.95 USD per Euro. The US rate of interest is 6%, but in Europe it is 5%. The question is, what is the fair rate of exchange on the forward market?

Solving for this rate is much easier than option pricing:

Forward Exchange = 0.95 x ((1+.06)/(1+.05), since in Europe the US is the foreign market.

The rate would be 0.9590.

This means that the market feels the Euro will strengthen against the dollar, since it is predicting a higher exchange rate in dollar terms for the Euro in the immediate future (1 year from now).

Discussion Questions

1. Why do you think people invest in options?

2. What is the difference between a natural hedge and an option?

3. What are some of the risks associated with investing in a foreign country?